Exercises

E8.102 [2BP] Prerequisites: [OKK], [OMM], [OK4]. Difficulty: *. Let Ω be an infinite uncountable set ; consider $X = \mathbb{R}^\Omega$ with the topology τ seen in [OMM].

1. Show that every point in (X, τ) does not admit a countable fundamental system of neighborhoods.
2. Setting
$$C \stackrel{\text{def}}{=} \{f \in X, f(x) \neq 0 \ \text{ for at most countably many } \ x \in \Omega\} \tag{8.103}$$
show that $\overline{C} = X$;
3. and that if $(f_n) \subset C$ and $f_n \to f$ pointwise then $f \in C$.
4. Let I be the set of all finite subsets of Ω, this is a filtering set if sorted by inclusion; consider the net
$$\varphi : I \to X \quad , \varphi(A) = \mathbb{1}_A$$
then $\forall A \in I, \varphi(A) \in C$ but
$$\lim_{A \in I} \varphi(A) = \mathbb{1}_X \notin C \quad .$$

Solution 1. [2BQ]