Definition 8.e.2. [2BR]*Let (X,τ) be a topological space. Given $A,B\subseteq X$, to shorten the formulas we will use the (nonstandard) notation*

- $A\mathbf{i}B$ *to say that A,B have non-empty intersection,*
- $A\mathbf{d}B$ *to say that they are disjointed, and*
- $\mathbf{n}A$ *to say that A it is not empty.*

we recall the definition of connectedness *(Chap. 5 Sec. 11 of the notes [**?**] or, Chap. 2 in [**?**]).*

- *The space X is disconnected if it is the disjoint union of two open non-empty sets.*
- *The space X is connected if it is not disconnected. This may be rewritten in different fashions, as for example*

$$\forall A,B\in\tau, (\mathbf{n}A \ \wedge \ \mathbf{n}B \ \wedge \ X\subseteq A\cup B)\Rightarrow A\mathbf{i}B\ .$$

- *A non-empty subset $E\subseteq X$is disconnected if it is disconnected with the induced topology; that is, if E is covered by the union of two open sets, each of which intersects E, but which are disjointed in E; in symbols,*

$$\exists A,B\in\tau, E\mathbf{i}A \ \wedge \ E\mathbf{i}B \ \wedge \ E\subseteq A\cup B \ \wedge \ A\cap B\cap E=\emptyset\ . \quad (8.e.3)$$

- *Similarly a non-empty set $E\subseteq X$ is connected if it is connected with the induced topology. This may be written as*

$$\forall A,B\in\tau, (E\mathbf{i}A \wedge E\mathbf{i}B \wedge E\subseteq A\cup B)\Rightarrow A\cap B\cap E\neq\emptyset\ . \quad (8.e.4)$$

or equivalently

$$\forall A,B\in\tau, (E\subseteq A\cup B\wedge A\cap B\cap E=\emptyset)\Rightarrow (E\subseteq A\vee E\subseteq B)\ . \quad (8.e.5)$$