11.2 Isometries [2CH]

We rewrite the definition [0TK] in the case of normed spaces.

Definition 11.22. *[110]*

We will compare it with this definition.

Definition 11.23. *[111]*

If φ is linear then the definition of equation [(11.24)] is equivalent to the definition of *linear isometry* seen in equation [(11.26)] (just set $z = x - y$). This explains why both are called ”isometries”.

By the Mazur–Ulam theorem [**?**] if M_1, M_2 are vector spaces (on real field) equipped with norm and φ is a surjective isometry, then φ is affine (which means that $x \mapsto \varphi(x) - \varphi(0)$ is linear).

We now wonder if there are isometries that are not linear maps, or more generally affine maps.

Exercises

E11.24 [112]

E11.25 [114]