

Remark 6.2. [2JJ] Given a set $I \subset \mathbb{R}$ there are various ways of saying that a function $f : I \rightarrow \mathbb{R}$ is **monotonic**. Let's first list the different types of monotonicity

$$\forall x, y \in I, x < y \implies f(x) \leq f(y) \quad (6.3)$$

$$\forall x, y \in I, x < y \implies f(x) < f(y) \quad (6.4)$$

$$\forall x, y \in I, x < y \implies f(x) \geq f(y) \quad (6.5)$$

$$\forall x, y \in I, x < y \implies f(x) > f(y) \quad (6.6)$$

Unfortunately in common use there are different and incompatible conventions used when naming the previous definitions. Here is a table, in which every convention is a column.

(6.3)	<i>non-decreasing</i>	<i>increasing</i>	<i>weakly increasing</i>
(6.4)	<i>increasing</i>	<i>strictly increasing</i>	<i>strictly increasing</i>
(6.5)	<i>non-increasing</i>	<i>decreasing</i>	<i>weakly decreasing</i>
(6.6)	<i>decreasing</i>	<i>strictly decreasing</i>	<i>strictly decreasing</i>

In this text, the convention in the last column is used.

(The first column is, in my opinion, problematic. It often leads to the use, unfortunately common, of phrases such as "*f is a non-decreasing function*" or "*we take a function f not decreasing*"; this can give rise to confusion: seems to say that *f* does not meet the requirement to be "*decreasing*", but it does not specify whether it is monotonic. People who follow the convention in the first column (in my opinion) should always say "*monotonic*").