

Remark 3.j.26. [2F2] Cantor proved that $|\mathbb{N}| < |\mathbb{R}|$. Cantor then (in 1878) formulated the continuum hypothesis *CH*: for any infinite set $E \subseteq \mathbb{R}$, either $|E| = |\mathbb{R}|$ or $|E| = |\mathbb{N}|$. For many year mathematicians tried to prove (or disprove) *CH*. It took decades to understand that this was not possible. We know know that, if *ZF* is consistent, then neither *CH* nor its negation can be proven as theorems in *ZF* (even using the Axiom of Choice). The second part of the statement was proved by Gödel nel 1939. The first part by Cohen in 1963. See Chap. 6 in [?].