Exercises

E10.2.44 [2F3]Topics:perfect set.Prerequisites:[0QP],[2F2],[2FD].Difficulty:

Suppose (X, d) is a complete metric space. A closed set $E \subseteq X$ without isolated points, *i.e.* consisting only of accumulation points, is called a **perfect set**.

Let C be the Cantor set. Assume that E is perfect and non-empty. Show that there exists a continuous function $\varphi : C \to E$ that is an homeomorphism with its image. This implies that $|E| \geq |\mathbb{R}|$.

So, in a sense, any non-empty perfect set contains a copy of the Cantor set.

This can be proven without relying on continuum hypothesis *[2F2]*. Cf. *[0W3]*.

Due to [0J8], it is enough to show that there exists a $\varphi : C \to E$ continuous and injective.

Solution 1. *[2F4]*