

Exercises

E8.h.15 [2F5] Consider a totally ordered set X (that has at least two elements), and the family \mathcal{F} of all open-ended intervals

$$\begin{aligned}(x, \infty) &\stackrel{\text{def}}{=} \{z \in X : x < z\} , & (-\infty, y) &\stackrel{\text{def}}{=} \{z \in X : z < y\} , \\ & & (x, y) &\stackrel{\text{def}}{=} \{z \in X : x < z < y\}\end{aligned}\tag{8.h.16}$$

for all $x, y \in X$. (Cf. [07D].) Prove that this is a base for a topology, *i.e.* that it satisfies [0KZ]. So \mathcal{F} is a base for the topology τ that it generates. This topology τ is called **order topology**.

If X has no maximum and no minimum, then only the intervals (x, y) are needed to form a base for τ . This is the case for the standard topologies on $\mathbb{R}, \mathbb{Q}, \mathbb{Z}$,