Exercises

E8.91 [2F7] Prerequisites: [OM3], [OM5], [OM7]. Let, more in general, I be a non-empty index set, and let (X_i, τ_i) be topological spaces, for $i \in I$; let $\mathcal{B}_i$ be a base for τ_i. (Note that the choice $\mathcal{B}_i = \tau_i$ is allowed.)

Let $X = \prod_{i \in I} X_i$ be the Cartesian product.

We define the *product topology* τ on X, similarly to [OM3], but with a twist.

A base $\mathcal{B}$ for τ is the family of all sets of the form $A = \prod_{i \in I} A_i$ where

$$\forall i \in I, A_i \in \mathcal{B}_i \vee A_i = X_i \quad ,$$

and moreover $A_i = X_i$ but for finitely many i.

Show that $\mathcal{B}$ satisfies the requirements in [OKX], so it is a base for the topology τ that it generates. Show that the product topology does not depend on the choice of the bases $\mathcal{B}_i$.

Solution 1. [2F8]