Exercises

E8.h.18 [2F9]Prerequisites:[2F5],[2F7],[071],[2F7].

Consider totally ordered sets $(X_i, \le_i)$ (each has at least two elements), and the associated *order topologies* τ_i.

Let $I = \mathbb{N}$ or $I = \{0, 1, \dots N\}$; let $X = \prod_{i \in I} X_i$ be the Cartesian product.

Consider these two topologies.

- We define the *product topology* τ on X, as explained in [2F7].
- We order X with the lexicographical order $\preceq$, and then we build the order topology σ on X. (See [071],[2F7])

Is there an inclusion between σ and τ?

If every X_i is finite, prove that these two topologies coincide [a].

Solution 1. *[2FC]*

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[a] Note that the order topology on a finite set is also the discrete topology; use [2FD].