The Scuola Normale Superiore (SNS) is a prestigious university institution with special status that welcomes students into two distinct paths: the undergraduate course (parallel to undergraduate and master's degree programs) and the advanced course (PhD).

SNS students in the undergraduate course are required to take some "internal courses" during the academic year, in addition to their regular university courses (to which they are duly enrolled at the University of Pisa). Every year, first-year SNS students in subjects such as Mathematics, Physics, Chemistry, and Biology have followed an annual internal Mathematics course. This course aims to delve deeper into and expand upon the traditional concepts included in the curricula of university courses that SNS students simultaneously attend.

Over the last 15 years, this course has covered several fundamental topics. It begins with a more thorough treatment of the foundations of Mathematics, including set theory based on the Zermelo-Fraenkel axioms, the construction of the set of natural numbers, and the characterization of real numbers as a complete ordered field. It then progresses to topics such as series and sequences, metric spaces and topology, differential calculus, and ordinary differential equations.

In these years, professors Giuseppe Da Prato, Fulvio Ricci, Luigi Ambrosio and Franco Flandoli have held the course. In addition to the author of this volume, Francesco Bonsante, Carlo Mantegazza, Simone Di Marino, Tommaso Pacini, Luciano Mari, Lorenzo Mazzieri, Andreas Hochenegger, Andrea Ferraguti, Alessandra Caraceni collaborated with TA.

The course notes have been published in [3].

The author has collaborated as TA, for more than ten years, accumulating a significant amount of theoretical material and exercises, which are now presented in this volume.

As is the case for text [3], this volume is not entirely self-contained as it is intended as a supplement to standard university courses in the first year. However, the first part is an exception because courses covering topics in logic fundamentals are not typically offered in the first year, and the texts used are often not written in a readily accessible language for first-year students. Therefore, Chapters 3 and 4 have been expanded to include the necessary theoretical elements, often disguised as exercises. Starting from Chapter 5, useful references to tackle the exercises are provided, along with some definitions and lemmas.

It should be noted that the numbering system in this volume follows a specific method: sections (and subsections), footnotes, and figures are numbered independently, while everything else in the volume follows a unique numbering system, divided by sections. This includes theorems, propositions, lemmas, equations, and more. The different numberings are made distinguishable by the use of Roman or Arabic numerals and/or special symbols, such as § for sections and † for notes.