

Exercises

E0.1 [2GT] Difficulty:*. Fix $n \in \mathbb{N}, n \geq 1$. Let $a_1, \dots, a_n \in \mathbb{R}$: then

$$\begin{aligned} \int_0^{a_1} \int_0^{a_2} \cdots \int_0^{a_n} \cos(x_1 + x_2 + \cdots x_n) \, dx_1 \, dx_2 \cdots \, dx_n &= \\ &= 2^n \cos\left(\frac{\sum_{i=1}^n a_i}{2}\right) \prod_{i=1}^n \sin\left(\frac{a_i}{2}\right) \end{aligned}$$

$$\begin{aligned} \int_0^{a_1} \int_0^{a_2} \cdots \int_0^{a_n} \sin(x_1 + x_2 + \cdots x_n) \, dx_1 \, dx_2 \cdots \, dx_n &= \\ &= 2^n \sin\left(\frac{\sum_{i=1}^n a_i}{2}\right) \prod_{i=1}^n \sin\left(\frac{a_i}{2}\right) \end{aligned}$$